Naval Research Laboratory
Washington, DC 20375-5000

NRL Memorandum Report 6471

Some Design Aspects of a Special Shipboard Array Antenna System

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September 19, 1989

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NRL Memorandum Report 6471	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Naval Research Laboratory 6b. OFFICE SYMBOL (If applicable) 5312	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Washington, DC 20375-5000	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Office of Naval Technology 8b. OFFICE SYMBOL (If applicable) OCNR	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code) Arlington, VA 22217-5000	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 6211N PROJECT NO. RA11A12 TASK NO. WORK UNIT ACCESSION NO. DN020-051

11. TITLE (Include Security Classification)
Some Design Aspects of a Special Shipboard Array Antenna System

12. PERSONAL AUTHOR(S)
Samaddar, S. N.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Interim	FROM ______ TO ______	1989 September 19	16

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Phased array antenna Broadband antenna radar

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This describes an analytical design of a special antenna system comprising one horn illuminating a planar array, each element of which is connected through an amplifier to the corresponding element of a second planar array, which radiates. The primary objective here is to make the total path length from the horn to any element of the second array via the respective intermediate points described above, the same for all such elements. This characteristics is very useful in making the system broadband. An example is given and a corresponding model having the equal path length characteristic is made.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL S. N. Samaddar	22b. TELEPHONE (Include Area Code) (202) 767-2584 22c. OFFICE SYMBOL 5312

DD Form 1473, JUN 86 Previous editions are obsolete. SECURITY CLASSIFICATION OF THIS PAGE

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SOME DESIGN ASPECTS OF A SPECIAL SHIPBOARD ARRAY ANTENNA SYSTEM

1. INTRODUCTION

In addition to the usual technical requirements, a shipborne electronically scanned phased array antenna system demands compactness for the obvious reason of economizing space, without sacrificing, however, the convenience of accessibility to the various parts of the system for their maintenance and operation. Moreover, special attention should be paid to the layout of different equipment so that it is cost effective. A special system using solid state amplifier with an emphasis of the above qualities has been described in reference [1]. In this paper we examine the geometry of a shipboard phased array system having a feature which enables the system's bandwidth to increase.

The antenna system considered here consists of a horn antenna, two planar array apertures and a specially contrived structure for housing amplifiers and associated hardwares. The latter structure will be called the amplifier-array, the surface of which is not known. The two planar array apertures are identical in all respects and they will be designated as array no. 1 and array no. 2, respectively. Array no. 1 lies in front of the horn antenna and receives an electromagnetic signal from this horn antenna (Fig. 1). The received signal from an element of array no. 1 is then fed via a cable to an amplifier. The amplified signal is then connected via a cable to an appropriate element of array no. 2, which is the transmitting aperture. In this way, an element of array no. 1 is connected to an appropriate element of array no. 2 via a particular amplifier. Similarly, other elements of arrays no. 1 and no. 2 are connected via separate amplifiers. Each of the planar arrays (no. 1 and no. 2) makes a small angle with the vertical. However, the angle between array no. 1 and array no. 2 may be less than or equal to 90°. The purpose of this study is to determine the positions of amplifiers in such a way that the total path length from the horn antenna to an element of array no. 2 via an appropriate element of array no. 1 and an amplifier is the same for all such similar paths. In other words, paths such as $PP'_1\, P'_2\, P'_3$ and $PP''_1\, P''_2\, P''_3$ are equal (Fig. 1). In view of the equal path lengths thus described, the array antenna system will have broadband characteristics [1], which are desirable features for some shipboard applications.

2. ANALYSIS

It is assumed that the orientations and other parameters (such as spacings, number of elements, etc.) of array no. 1 and no. 2 are given. In addition, the location of the horn antenna with respect to array no. 1 is also assumed to be known. As a result, the coordinates of the positions of elements of array no. 1 and array no. 2, together with the coordinates of the horn antenna, can be found in principle. The determination of these coordinates then becomes a problem of solid analytical geometry. Since we need these coordinates, let us first determine them as follows:

The lowest row of the elements of array no. 1 is aligned with the axis of x of a right-handed rectangular coordinate system x, y and z, of which z is along the vertical direction. If $\pi/2 - \theta_{01} > 0$ is the angle (acute) between the normal to the plane of array no. 1 and vertical z-axis, then the equation of the plane passing through array no. 1 is

$$y \cos \theta_{01} = z \sin \theta_{01}, \text{ for a given } x. \tag{1}$$

Manuscript approved June 1, 1989.

With reference to Fig. 1, $B_b C_B$ lies along the x-axis and θ_{01} is the angle between $C_b C$ and the z-axis. Let us now assume that the plane containing array no. 2 (i.e. the plane of array no. 2) intersects the x-axis at $x = -d'$ and makes an angle $\psi \leq \pi/2$ with the x-axis. If the normal to the plane of array no. 2 makes an angle $\pi/2 - \theta_{02} > 0$ with the vertical, then the equation of this plane (containing array no. 2) becomes

$$-x \sin \psi \sin \theta_{02} + y \cos \psi \sin \theta_{02} + z \sin. \theta_{02} = d' \sin \psi \sin \theta_{02} \tag{2}$$

In Fig. 1, D is the point of intersection betwen the x-axis and the line of intersection $A_b \bar{B}_b$ of the plane of array no. 2 and the x-y-plane. Then $Q_{1,1}^{(1)} D = -d'$, $\psi = \bar{B}_b D B_b$ and θ_{02} is the angle between $\bar{B}_b B$ and the z-axis.

Assume further that the lowest row of the elements of array no. 2 lies along $A_b \bar{B}_B$ in the horizontal $x - y$ plane. Let the distance of the nearest element, $Q_{1,N}^{(2)}$, lying in the $x - y$ plane, of array no. 2 from the origin be d_1. The direction of $Q_{1,N}^{(2)}$ from the origin makes an angle $\pi - \psi_1$ with the x-axis. Then it can be shown that the coordinates of this nearest element, $Q_{1,N}^{(2)}$ from the origin are (Fig. 1).

$$(-d_1 \cos \psi_1, d_1 \sin \psi_1, 0) \tag{3}$$

Note that $Q_{1,N}^{(2)} Q_{1,1}^{(1)} D = \psi_i$ and the distance $Q_{1,N}^{(2)} Q_{1,1}^{(1)} = d_1$, if it is assumed $Q_{1,1}^{(1)}$ is at the origin (Fig. 1).

The element, $Q_{1,N}^{(2)}$, whose coordinates are given by (3) is also the last element of lowest row of the array no. 2. Let a be the spacing between the consecutive elements parallel to the $x - y$ plane and b be the spacing in the other direction for both arrays. Since the normal to the plane of array no. 2 makes an angle $\pi/2 - \theta_{02} > 0$ with the vertical, then employing some tedious procedures of 3-dimensional analytical geometry, the coordinates of a general element $Q_{i,\ell}^{(2)}$ of array no. 2 can be written in the following form.

$$x_{i,\ell}^{(2)} = -d_1 \cos \psi_1 + (i - 1) b \sin \psi \sin \theta_{02} + (N - \ell)\, a \cos \psi, \tag{4i}$$

$$y_{i,\ell}^{(2)} = d_1 \sin \psi_1 - (i - 1) b \cos \psi \sin \theta_{02} + (N - \ell)\, a \sin \psi, \tag{4ii}$$

$$z_{i,\ell}^{(2)} = (i - 1) b \cos \theta_{02}. \tag{4iii}$$

The elements of array no. 1 are also arranged in a similar fashion as those of array no. 2. For finding the coordinates of the elements of array no. 1, let us assume that the first element denoted by $Q_{i,\ell}^{(1)}$ of this array is located at the origin of the rectangular coordinate system x, y, z used here. Then the coordinates of the general element $Q_{i,\ell}^{(1)}$ of array no. 1 can be shown to have the following representations (See Eq. 1).

$$x_{i,\ell}^{(1)} = (\ell - 1) a, \tag{5i}$$

$$y_{i,\ell}^{(1)} = (i - 1) b \sin \theta_{01}, \tag{5ii}$$

$$z_{i,\ell}^{(1)} = (i - 1) b \cos \theta_{01}, \tag{5iii}$$

$$i, \ell = 1, 2, \ldots, N$$

Let r_o be the distance of the horn antenna at P from the midpoint $Q^{(1)}_{i,(N+1)/2}$ of the bottom (lowest) row of array no. 1. Assume further that the line $PQ^{(1)}_{1,(N+1)/2}$ makes and angle $-\hat{\theta}_0$ with the horizontal plane. Then the coordinates of the position P of the horn antenna are given by

$$x_0 = (N - 1)a/2, \tag{6i}$$

$$y_0 = -r_0 \cos \hat{\theta}_0, \tag{6ii}$$

$$z_0 = -r_0 \sin \hat{\theta}_0, \tag{6iii}$$

A given element $Q^{(1)}_{i,j}$ of array no. 1 will be connected by a cable to the element $Q^{(2)}_{i,j}$ of array no. 2 via a suitably positioned amplifier unit at $Q^{(3)}_{i,j}$. The coordinates of the position $Q^{(3)}_{i,j}$ denoted by $x^{(3)}_{i,j}$, $y^{(3)}_{i,j}$, and $z^{(3)}_{i,j}$ will be determined subject to the following condition.

$$|PQ^{(1)}_{i,j}| + |Q^{(1)}_{i,j}\, Q^{(3)}_{i,j}| + |Q^{(3)}_{i,j}Q^{(2)}_{i,j}| = \text{a constant}, \tag{7}$$

for all values of i and $j = 1, 2, \ldots, N$, where $|PQ^{(1)}_{i,j}|$ is the distance between the point P and the element at $Q^{(1)}_{i,j}$. Similar meaning for other quantities in (7). The superscript 3 is associated with amplifier positions.

The positions of N^2 number of amplifiers are not known. Since each position is determined by 3 coordinates, there are $3N^2$ unknowns. The relation (7) provides only $(N^2 - 1)$ equations. Thus we have a situation where number of unknown exceeds the number of equations. Therefore, the positions of the amplifiers cannot be determined uniquely. Since the only condition which one shall have to satisfy is given by (7), one may introduce other conditions in a convenient manner so that the structure on which the amplifier units will be mounted can be constructed readily. For instance, if one chooses $a = b$ and $\theta_{01} = \theta_{02}$, then the i-th row of array no. 1 and the i-th row of array no. 2 will lie on a horizontal plane, on which the corresponding amplifier units can be mounted. In view of such an arrangement, the condition (7) dictates that the position of the amplifier $Q^{(3)}_{i,j}$ which is connected to the elements $Q^{(1)}_{i,j}$ and $Q^{(2)}_{i,j}$ will lie on an ellipse whose foci are at $Q^{(1)}_{i,j}$ and $Q^{(2)}_{i,j}$. This ellipse will, of course, lie on the horizontal plane passing through the i-th row of both array no. 1 and array no. 2. Although this arrangement will reduce the number of unknowns considerably, the position $Q^{(3)}_{i,j}$ (of an amplifier unit) cannot still be determined uniquely, since $Q^{(3)}_{i,j}$ can be anywhere on the ellipse. Therefore, additional suitably chosen conditions must be imposed in order to determine uniquely the position of the amplifier $Q^{(3)}_{i,j}$, with $i, j = 1, 1, \ldots, N$. This will be clearly demonstrated through the following example.

3. AN EXAMPLE

$$N = 11 = \text{number of elements in a row and a column},$$

$$a = b = 2 \text{ cm} = \text{spacing between consecutive elements},$$

$$r_0 = (N - 1)b = 20 \text{ cm},$$

$$\theta_{01} = \theta_{02} = 15^\circ = \text{elevation of the planes of array no. 1 and array no. 2 with vertical},$$

$$\psi_1 = 45°,$$

$$\hat{\theta}_0 = \sin^{-1}(\cdot 1),$$

$$d' = (N - 1)b \sin \theta_{01} = 5.176 \text{ cm},$$

$$d_1 \cos \psi_1 = d_1 \sin \psi_1 = 5.176 \text{ cm},$$

$$(N - 1)b \cos \theta_{01} = 19.3186 \text{ cm},$$

$$x_0 = (N - 1)a/2 = 10.0 \text{ cm},$$

$$y_0 = -r_0 \cos \hat{\theta}_0 = -19.90 \text{ cm}$$

$$z_0 = r_0 \sin \theta_0 = -2.0 \text{ cm}.$$

For the sake of the economy of space coordinates of the three rows (bottom, middle and top) of array no. 1 and array no. 2 will be computed and presented here. The coordinates of the elements in the bottom row of the array no. 1 are given by (See Eq. 5)

$$Q_{i,j}^{(1)} = [x_{1,j}^{(1)}, y_{1,j}^{(1)}, z_{1,j}^{(1)}], \quad i = 1$$

$$j = 1, 2, \ldots, 11 \tag{6}$$

$$x_{1,j}^{(1)} = 2(j - 1) \text{ cm},$$

$$y_{1,j}^{(1)} = 0 = z_{1,j}^{(1)}, \text{ for all } j$$

The coordinates of the elements in the center row of array no. 1 are given by

$$Q_{6,j}^{(1)} = [s_{6,j}^{(1)}, y_{6,j}^{(1)}, z_{c,j}^{(1)}], \quad i = 6$$

$$j = 1, 2, \ldots, 11$$

$$x_{6,j}^{(1)} = 2(j - 1) \text{ cm} \tag{7}$$

$$y_{6,j}^{(1)} = 2.588 \text{ cm}, \text{ for all } j$$

$$z_{6,j}^{(1)} = 9.659 \text{ cm}, \text{ for all } j$$

The coordinates of the elements in the top row of array no. 1 are as follows:

$$Q_{11,j}^{(1)} = [x_{11,j}^{(1)}, y_{11,j}^{(1)}, z_{11,j}^{(1)}]$$

$$i = 11$$

$$j = 1, 2, \ldots, 11$$

$$x^{(1)}_{11,j} = 2(j - 1) \text{ cm}$$

$$y^{(1)}_{11,j} = 5.176 \text{ cm}, \quad \text{for all } j \tag{8}$$

$$z^{(1)}_{11,j} = 19.318 \text{ cm}, \quad \text{for all } j$$

The coordinates of the elements in the bottom row of array no. 2 are given by (See Eq. 4)

$$Q^{(2)}_{1,j} = [x^{(2)}_{1,j}, y^{(2)}_{1,j}, z^{(2)}_{1,j}]$$

$$i = 1$$

$$j = 1, 2, \ldots, 11$$

$$x^{(2)}_{1,j} = -5.176 \text{ cm}, \quad \text{for all } j \tag{9}$$

$$y^{(2)}_{1,j} = [5.176 + 2(11 - j)] \text{ cm},$$

$$z^{(2)}_{1,j} = 0, \quad \text{for all } j$$

The coordinates of the elements in the center row of array no. 2 are as follows:

$$Q^{(2)}_{6,j} = [x^{(2)}_{6,j}, y^{(2)}_{6,j}, z^{(2)}_{6,j}]$$

$$i = 6$$

$$j = 1, 2, \ldots, 11 \tag{10}$$

$$x^{(2)}_{6,j} = -2.588 \text{ cm}, \quad \text{for all } j$$

$$y^{(2)}_{6,j} = [5.176 + 2(11 - j)] \text{ cm}$$

$$z^{(2)}_{6,j} = 9.659 \text{ cm}, \quad \text{for all } j$$

The coordinates of the elements in the top row of array no. 2 are given by

$$Q^{(2)}_{11,j} = [x^{(2)}_{11,j}, y^{(2)}_{11,j}, z^{(2)}_{11,j}]$$

$$i = 11$$

$$j = 1, 2, \ldots, 11$$

$$x^{(2)}_{11,j} = 0, \quad \text{for all } j \tag{11}$$

$$y^{(2)}_{11,j} = [5.176 + 2(11 - j)] \text{ cm}$$

$$z^{(2)}_{11,j} = 19.318 \text{ cm.}$$

In order to choose a fixed path length from the transmitter at P to $Q^{(2)}_{i,j}$ via $Q^{(1)}_{i,j}$ and the amplifier position $Q^{(3)}_{i,j}$, it is first necessary to calculate the possible minimum longest distance, which in this case is from P to $Q^{(2)}_{11,11}$ via $Q^{(1)}_{11,11}$ without taking the position of the amplifier into consideration. Then the path length $|PQ^{(1)}_{11,11}\ Q^{(2)}_{11,11}|$ can be expressed as

$$|PQ^{(1)}_{11,11}| + |Q^{(1)}_{11,11}\ Q^{(2)}_{11,11}|$$

$$= [\{x_0 - x^{(1)}_{11,11}\}^2 + \{y_o - y^{(1)}_{11,11}\}^2 + \{z_o - z^{(1)}_{11,11}\}^2]^{1/2} \tag{12}$$

$$+ [\{x^{(1)}_{11,11} - x^{(2)}_{11,11}\}^2 + \{y^{(1)}_{11,11} - y^{(2)}_{11,}\}^2]^{1/2}$$

$$= 54.40 \text{ cm.}$$

Since in the actual situation the above path must pass through the amplifier $Q^{(3)}_{11,11}$, the x and y-coordinates of which are yet to be determined, the path length $|PQ^{(1)}_{11,11}\ Q^{(3)}_{11,11}\ Q^{(2)}_{11,11}|$ will be longer than 54.40 cm. Let us then arbitrarily choose this length to be 60 cm, i.e.,

$$|PQ^{(1)}_{i,j}\ Q^{(3)}_{i,j}\ Q^{(2)}_{i,j}| = |PQ^{(1)}_{11,11}\ Q^{(3)}_{11,11}\ Q^{(2)}_{11,11}| = 60 \text{ cm}, \tag{13}$$

for all i and j.

Since the position of the amplifier $Q^{(3)}_{i,j}$ lies on an ellipse with foci at $Q^{(1)}_{i,j}$ and $Q^{(2)}_{i,j}$, the lengths $|Q^{(1)}_{i,j}\ Q^{(3)}_{i,j}\ Q^{(2)}_{i,j}|$ then must be determined. Using (13) one finds that

$$|Q^{(1)}_{i,j}\ Q^{(3)}_{i,j}\ Q^{(2)}_{i,j}| = 60 \text{ cm} - |PQ^{(1)}_{i,j}| \tag{14}$$

Since the positions of P and $Q^{(1)}_{i,j}$ are known, the lengths $PQ^{(1)}_{i,j}$ can be computed easily and the results are as follows:

$$PQ^{(1)}_{1,1} = PQ^{(1)}_{1,11} = 22.361 \text{ cm} \tag{15i}$$

$$PQ^{(1)}_{1,2} = PQ^{(1)}_{1,10} = 21.541 \text{ cm} \tag{15ii}$$

$$PQ^{(1)}_{1,3} = PQ^{(1)}_{1,9} = 20.881 \text{ cm} \tag{15iii}$$

$$PQ^{(1)}_{1,4} = PQ^{(1)}_{1,8} = 20.396 \text{ cm} \tag{15iv}$$

$$PQ^{(1)}_{1,5} = PQ^{(1)}_{1,7} = 20.10 \text{ cm} \tag{15v}$$

$$PQ^{(1)}_{1,6} = 20.00 \text{ cm} \tag{15vi}$$

$$PQ^{(1)}_{6,1} = PQ^{(1)}_{6,11} = 27.233 \text{ cm} \tag{16i}$$

$$PQ^{(1)}_{6,2} = PQ^{(1)}_{6,10} = 26.564 \text{ cm} \tag{16ii}$$

$$PQ^{(1)}_{6,3} = PQ^{(1)}_{6,9} = 26.032 \text{ cm} \tag{16iii}$$

$$PQ^{(1)}_{6,4} = PQ^{(1)}_{6,8} = 25.645 \text{ cm} \tag{16iv}$$

$$PQ^{(1)}_{6,5} = PQ^{(1)}_{6,9} = 25.409 \text{ cm} \tag{16v}$$

$$PQ^{(1)}_{6,6} = 25.331 \text{ cm} \tag{16vi}$$

$$PQ^{(1)}_{11,1} = PQ^{(1)}_{11,11} = 34.400 \text{ cm} \tag{17i}$$

$$PQ^{(1)}_{11,2} = PQ^{(1)}_{11,10} = 33.871 \text{ cm} \tag{17ii}$$

$$PQ^{(1)}_{11,3} = PQ^{(1)}_{11,9} = 33.455 \text{ cm} \tag{17iii}$$

$$PQ^{(1)}_{11,4} = PQ^{(1)}_{11,8} = 33.155 \text{ cm} \tag{17iv}$$

$$PQ^{(1)}_{11,5} = PQ^{(1)}_{11,7} = 32.974 \text{ cm} \tag{17v}$$

$$PQ^{(1)}_{11,6} = 32.913 \text{ cm.} \tag{17vi}$$

Let us now compute the lengths $|Q^{(1)}_{i,j}\ Q^{(3)}_{i,j}\ Q^{(2)}_{i,j}|$ given by (14) using the values of $PQ^{(1)}_{i,j}$ presented in Eq. (15) to (17). The results are given in the following:

$$|Q^{(1)}_{1,1}\ Q^{(3)}_{1,1}\ Q^{(2)}_{1,1}| =$$

$$|Q^{(1)}_{1,11}\ Q^{(3)}_{1,11}\ Q^{(2)}_{1,11}| = 37.639 \text{ cm} \tag{18i}$$

$$|Q^{(1)}_{1,2}\ Q^{(3)}_{1,2}\ Q^{(2)}_{1,2}| =$$

$$|Q^{(1)}_{1,10}\ Q^{(3)}_{1,10}\ Q^{(2)}_{1,10}| = 38.459 \text{ cm} \tag{18ii}$$

$$|Q^{(1)}_{1,3}\ Q^{(3)}_{1,3}\ Q^{(2)}_{1,3}| =$$

$$|Q^{(1)}_{1,9}\ Q^{(3)}_{1,9}\ Q^{(2)}_{1,9}| = 39.119 \text{ cm} \tag{18iii}$$

$$|Q^{(1)}_{1,4}\ Q^{(3)}_{1,4}\ Q^{(2)}_{1,4}| = |Q^{(1)}_{1,8}\ Q^{(3)}_{1,8}\ Q^{(2)}_{1,8}| = 39.604 \text{ cm} \tag{18iv}$$

$$|Q^{(1)}_{1,5}\ Q^{(3)}_{1,5}\ Q^{(2)}_{1,5}| = |Q^{(1)}_{1,7}\ Q^{(3)}_{1,7}\ Q^{(2)}_{1,7}| = 39.90 \text{ cm} \tag{18v}$$

$$|Q^{(1)}_{1,6}\ Q^{(3)}_{1,6}\ Q^{(2)}_{1,6}| = 40.0 \text{ cm} \tag{18vi}$$

$$|Q^{(1)}_{6,1}\ Q^{(3)}_{6,1}\ Q^{(2)}_{6,1}| = |Q^{(1)}_{6,11}\ Q^{(3)}_{6,11}\ Q^{(2)}_{6,11}| = 32.767 \text{ cm} \tag{19i}$$

$$|Q^{(1)}_{6,2}\ Q^{(3)}_{6,2}\ Q^{(2)}_{6,2}| = |Q^{(1)}_{6,10}\ Q^{(3)}_{6,10}\ Q^{(2)}_{6,10}| = 33.436 \text{ cm} \tag{19ii}$$

$$|Q^{(1)}_{6,3}\ Q^{(3)}_{6,3}\ Q^{(2)}_{6,3}| = |Q^{(1)}_{6,9}\ Q^{(3)}_{6,9}\ Q^{(2)}_{6,9}| = 33.968 \text{ cm} \tag{19iii}$$

$$|Q^{(1)}_{6,4}\ Q^{(3)}_{6,4}\ Q^{(2)}_{6,4}| = |Q^{(1)}_{6,8}\ Q^{(3)}_{6,8}\ Q^{(2)}_{6,8}| = 34.356 \text{ cm} \tag{19iv}$$

$$|Q^{(1)}_{6,5}\ Q^{(3)}_{6,5}\ Q^{(2)}_{6,5}| = |Q^{(1)}_{6,7}\ Q^{(3)}_{6,7}\ Q^{(2)}_{6,7}| = 34.591 \text{ cm} \tag{19v}$$

$$|Q^{(1)}_{6,6}\ Q^{(3)}_{6,6}\ Q^{(2)}_{6,6}| = 34.669 \text{ cm} \tag{19vi}$$

$$|Q^{(1)}_{11,1}\ Q^{(3)}_{11,1}\ Q^{(2)}_{11,1}| = |Q^{(1)}_{11,11}\ Q^{(3)}_{11,11}\ Q^{(2)}_{11,11}| = 25.600 \text{ cm} \tag{20i}$$

$$|Q^{(1)}_{11,2}\ Q^{(3)}_{11,2}\ Q^{(2)}_{11,2}| = |Q^{(1)}_{11,10}\ Q^{(3)}_{11,10}\ Q^{(2)}_{11,10}| = 26.129 \text{ cm} \tag{20ii}$$

$$| Q^{(2)}_{11,3}\ Q^{(3)}_{11,3}\ Q^{(2)}_{11,3} | =$$

$$| Q^{(1)}_{11,9}\ Q^{(3)}_{11,9}\ Q^{(2)}_{11,9} | = 26.545 \text{ cm} \tag{20iii}$$

$$| Q^{(1)}_{11,4}\ Q^{(3)}_{11,4}\ Q^{(2)}_{11,4} | =$$

$$| Q^{(1)}_{11,8}\ Q^{(3)}_{11,8}\ Q^{(2)}_{11,8} | = 26.845 \text{ cm} \tag{20iv}$$

$$| Q^{(1)}_{11,5}\ Q^{(3)}_{11,5}\ Q^{(2)}_{11,5} | =$$

$$| Q^{(1)}_{11,7}\ Q^{(3)}_{11,7}\ Q^{(2)}_{11,7} | = 27.026 \text{ cm} \tag{20v}$$

$$| Q^{(1)}_{11,6}\ Q^{(3)}_{11,6}\ Q^{(2)}_{11,6} | = 27.087 \text{ cm}. \tag{20vi}$$

Note that the points $Q^{(1)}_{i,j}$, $Q^{(3)}_{i,j}$, $Q^{(2)}_{i,j}$ lie in a horizontal plane passing through the i-th row of array no. 1 and array no. 2. Since the length $| Q^{(1)}_{i,j}\ Q^{(3)}_{i,j}\ Q^{(2)}_{i,j} |$ is fixed as shown in Eqs. 18 to 20, the amplifier unit at $Q^{(3)}_{i,j}$ lie on an ellipse (drawn on the horizontal plane passing through the i-th row), having $Q^{(1)}_{i,j}$ and $Q^{(2)}_{i,j}$ as its foci. Although the point $Q^{(3)}_{i,j}$ lie on an ellipse which can be drawn easily, its exact location is yet to be determined uniquely. In order to accomplish this, additional suitable requirements must be imposed. For convenience, let us choose N to be an odd integer (which is the case in this example). Then $Q^{(1)}_{i,j}$, $Q^{(2)}_{i,j}$ and $Q^{(3)}_{i,j}$ (with $j = (N + 1)/2$) represent respectively the locations of two central elements in the i-th row of the two arrays and the corresponding central amplifier unit. Let us draw a perpendicular bisector of the segment joining the points $Q^{(1)}_{i,j}$ and $Q^{(2)}_{i,j}$ with $j = (N + 1)/2$. Then the position $Q^{(3)}_{i,j}$ with $j = (N + 1)/2$ will be taken as the point of intersection of this perpendicular bisector and the ellipse with foci at $Q^{(1)}_{i,j}$ and $Q^{(2)}_{i,j}$ with $j = (N + 1)/2$. Note that both the perpendicular bisector and this ellipse lie in the i-th horizontal plane, i.e., the horizontal plane passing through the i-th row of both the array no. 1 and array no. 2. For maintaining symmetry, let us also assume that the amplifier units $Q^{(3)}_{i,j}$ for $j > (N + 1)/2$ will lie between the perpendicular bisector and the i-th row of the array no. 1, whereas the amplifier units $Q^{(3)}_{i,j}$ with $j < (N + 1)/2$ will lie between the perpendicular bisector and the i-th row of array no. 2. Let us further assume that the spacing between two consecutive amplifier units in the i-th horizontal plane be a preselected length ρ. The method of selection of ρ will be described shortly. Since we have already determined the position $Q^{(3)}_{i,j}$ with $j = (N + 1)/2$ as described above, the adjacent location $Q^{(3)}_{i,j}$ with $j = (N + 3)/2$ will be the point of intersection between the arc of radius ρ centered at $Q^{(3)}_{i,j}$ with $j = (N + 1)/2$ and the ellipse having foci at $Q^{(1)}_{i,j}$ and $Q^{(2)}_{i,j}$ with $j = (N + 3)/2$. The next point $Q^{(3)}_{i,j}$ with $j = (N + 5)/2$ will be at the intersection of the arc of radius ρ centered at $Q^{(3)}_{i,j}$ with $j = (N + 3)/2$ and the ellipse having foci at $Q^{(1)}_{i,j}$ and $Q^{(2)}_{i,j}$ with $j = (N + 5)/2$. In this way, all the positions $Q^{(3)}_{i,j}$ of the amplifier units lying on the i-th horizontal plane can be determined uniquely. The process is repeated for other rows (i.e., different values of i).

The length ρ should be chosen in such a way that the amplifier units are not crowded together and at the same time, they are placed well within the quadrant bounded by the i-th row of array no. 1 and array no. 2. A convenient way to select ρ is to take $\rho = \sigma a$, where a is the spacing between the elements of both arrays and σ is a suitable constant. In. the present example, σ is chosen 1.25.

The above procedure of locating $Q^{(3)}_{i,j}$ uniquely is illustrated through Fig. 3, where array elements are chosen to be 7 $\times$ 7 for convenience. In Fig. 3, AC and AB represent top-row (i = 7) of array no. 1 and array no. 2 respectively. AA' is the perpendicular bisector of the segment $Q^{(2)}_{7,4}$

and $Q_{7,4}^{(1)}$. *BAC* is the top horizontal plane ($i = 7$). *PAC* is an inclined plane. ρ should be the same for all rows (or i). Figure 4 shows the picture of the model which is designed using the parameters computed in the example considered here.

4. CONCLUSION

The feasibility of designing a special shipboard array antenna system (with amplifier units) having equal path length feature has been demonstrated. An example is provided. A model is then built using the parameters computed in the example. Although amplifier units are placed on different horizontal planes (for different rows), a three dimensional surface can also be constructed in principle, for mounting all the amplifier units. Due to equal path length feature, this antenna system will have broadband characteristic.

It may be noted that this unique way of achieving equal path length permits ready access to the array apertures and all amplifiers.

5. ACKNOWLEDGMENT

This design problem was suggested by Mr. T. C. Cheston, with whom the author had several useful discussions. The author appreciates very much also the assistance provided by Mr. James B. Thomas, Jr., who built a model which proved the feasibility of the design described.

6. REFERENCE

1. T.C. Cheston, "Phased Array Lens Radar System With Shared Solid-State Modules," NRL Memorandum Report 4963, Nov. 26, 1982.

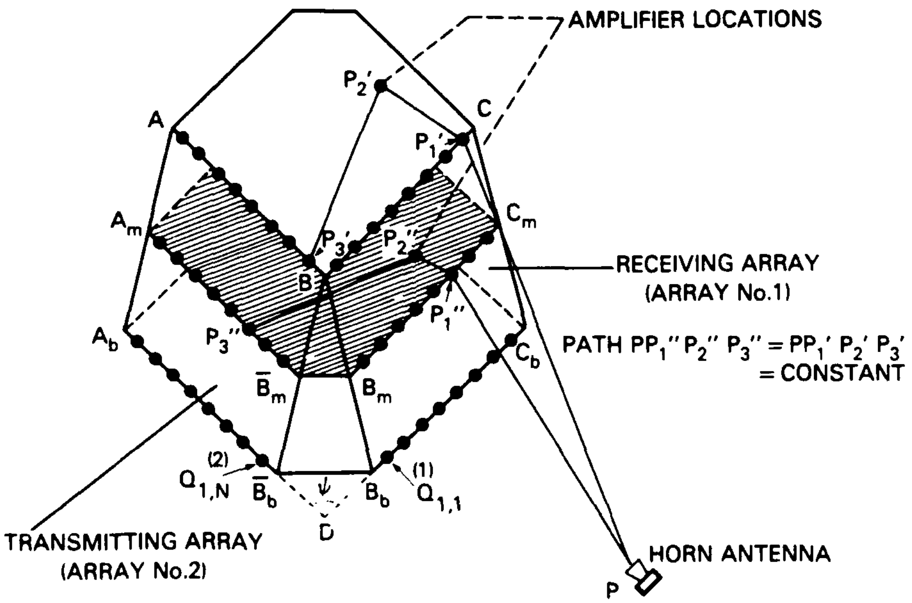

Fig. 1 — A sketch of two planar arrays, a horn antenna and amplifier locations

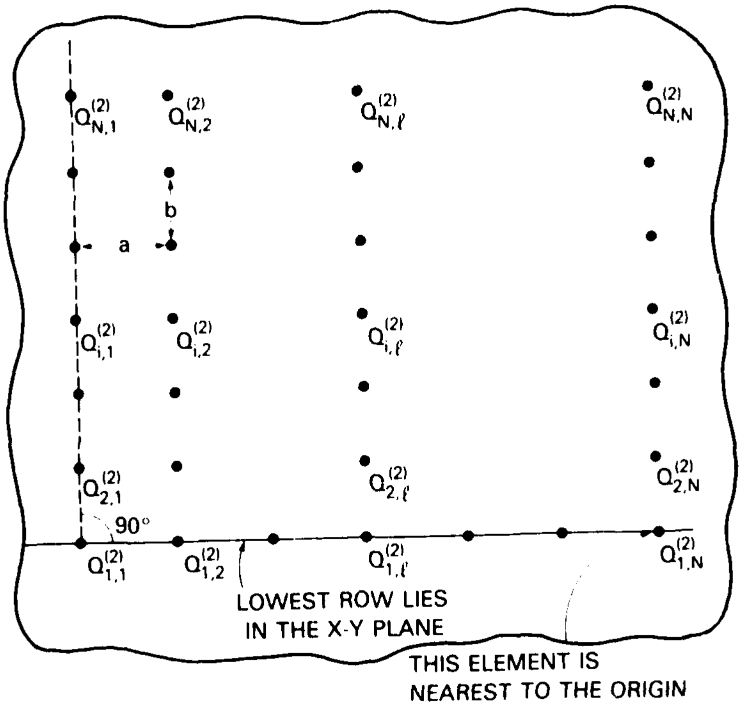

Fig. 2 — Locations of elements of the array no. 2 ($N \times N$ elements planar array)

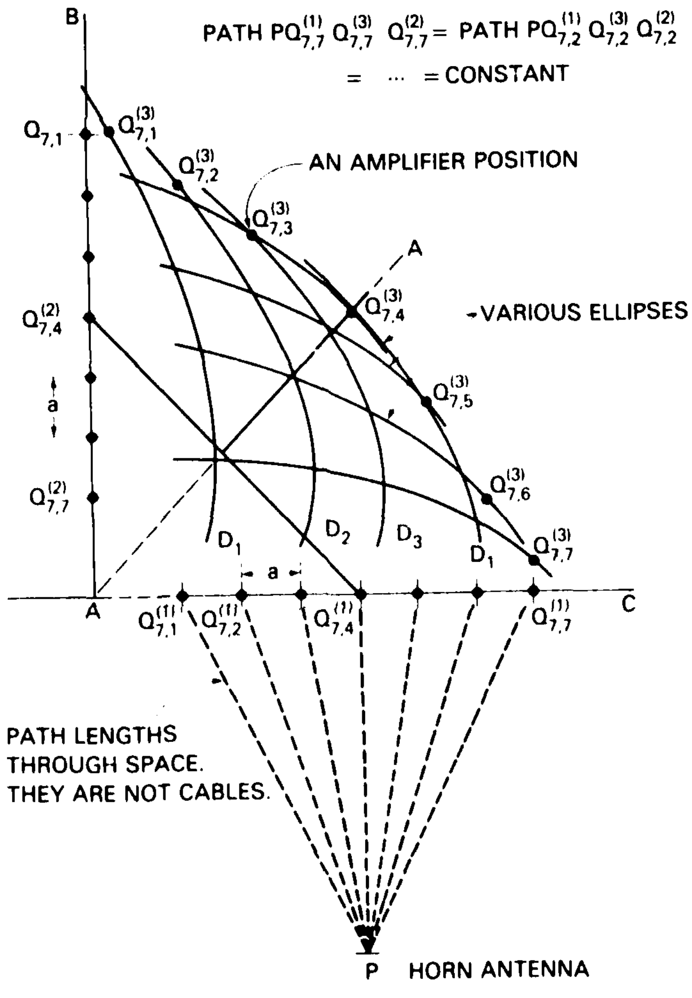

Fig. 3 — Principle of finding positions of amplifiers for a 7×7 elements of an array system

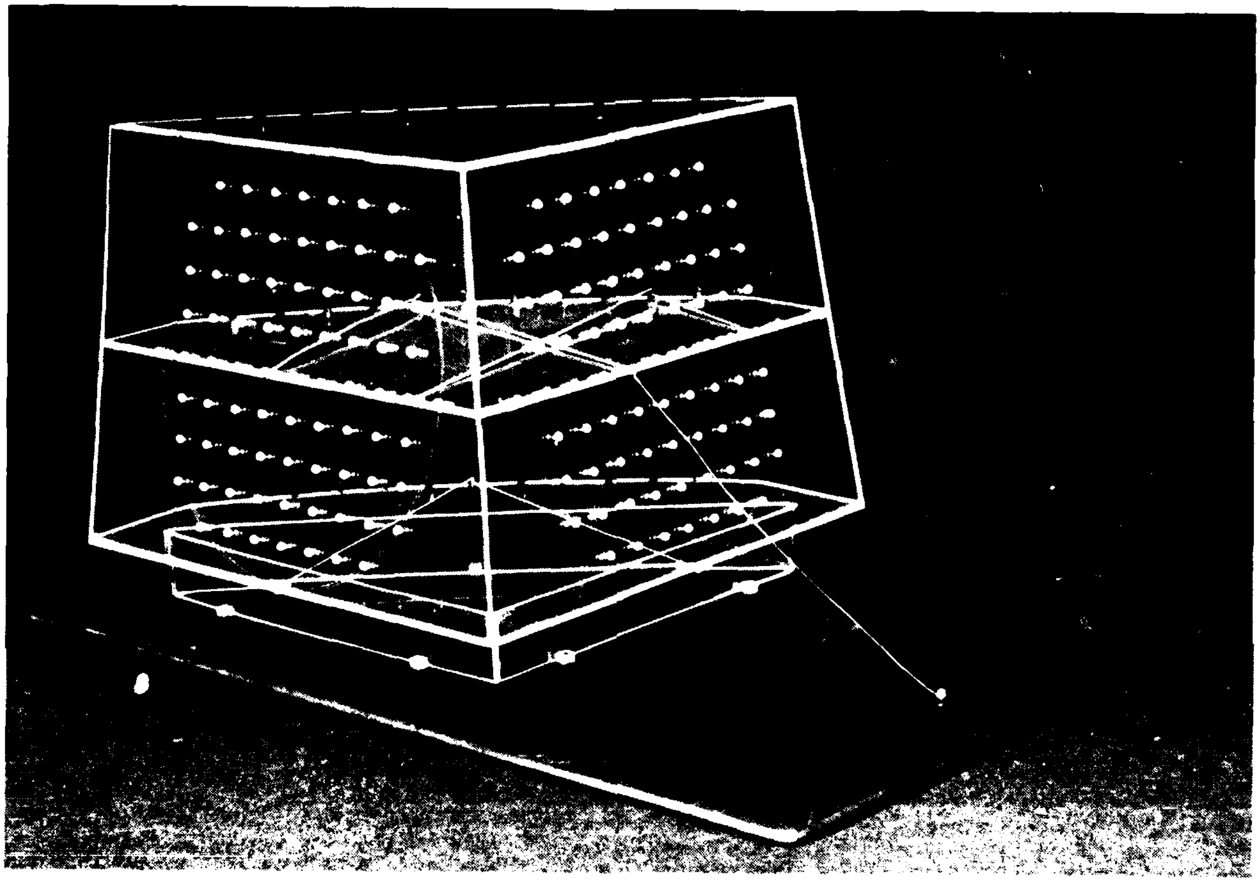

Fig. 4 - A model built for the verification of the analysis